NOTES

Measurement of Normal Stresses in Simple Shearing Flow of Ellis Fluid

Several methods[1-7] for measurement of normal stresses in simple shearing flow have been suggested. One of the methods is based on measurement of the swelling of a jet issuing from a capillary. Metzner et al.[4] have presented this method, which related normal stresses to jet diameters. They have analyzed a portion of the jet between section 1 and section 2 (Fig. 1 Ref. 4) by applying[8] Newton's Second Law, deriving eq. (1)

$$(t_{11} - t_{33})_R = \frac{\rho V^2}{n'} \left\{ (n' + 1) \int_0^1 2 \left(\frac{u}{V}\right)^2 \frac{r}{R}\, d\,(r/R) - \left(\frac{D}{d_j}\right)^2 \left[n' + 1 + \frac{d \log (D/d_j)}{d\,(\log (8V/D))} \right] \right\} \quad (1)$$

where t_{11}, t_{33} are normal stresses in the radial and axial directions, respectively, in the tube; ρ is the fluid density, n' is the flow behavior index defined by the logarithmic slope of the flow curve obtained from a capillary tube; u is the local velocity at the position defined by the radial coordinate r; V is the mean or volumetric fluid velocity inside the tube; R is the tube radius; D is the tube inside diameter; and d_j is the jet diameter.

where

$$n' = \frac{d \log (t_w)}{d \log (8V/D)}$$

and t_w is the point value of shear stress evaluated at the wall.

In order to evaluate the integral in eq. (1), it is necessary to know the velocity profile, and any relation like the power law or Ellis fluid model can be made applicable. Metzner et al. have chosen the power law equation.

In the present paper the velocity profile based on the Ellis fluid is made applicable to determine the normal stresses.

The equation for the Ellis fluid[9,10] is represented as

$$t_{21} = -\eta(V_1/dx_2) \quad (2)$$

where t_{21} is the shear stress and x_2 is the distance along the perpendicular to the axial coordinate, with

$$1/\eta = (1/\eta_0)(1 + |t_{21}/t_{1/2}|^{\alpha-1}) \quad (3)$$

where α is the Ellis fluid constant and $t_{1/2}$ is the value of shear stress t_{21} for which $\eta = \eta_0/2$.

From eq. (3) the velocity profile of the jet is given by

$$\begin{aligned} u &= \frac{Rt_w}{2\eta_0} \left\{ \left[1 - \left(\frac{r}{R}\right)^2 \right] + \frac{2}{\alpha + 1} \left(\frac{t_w}{t_{1/2}}\right)^{\alpha-1} [(1 - r/R)^{\alpha+1}] \right\} \\ &= K_1[1 - (r/R)^2] + K_2[1 - (r/R)^{\alpha+1}] \end{aligned} \quad (4)$$

where

$$K_1 = Rt_w/2\eta_0$$

and

$$K_2 = (Rt_w/2\eta_0)[2/(\alpha + 1)](t_w/t_{1/2})^{\alpha-1}$$

On substituting eq. (4) in eq. (1), the integral can be evaluated, giving

$$(t_{11} - t_{33}) = \frac{\rho V^2}{n'}\left\{\frac{n'+1}{V^2}\left[\frac{K_1^2}{3} + K_3 + K_4\right] - \left(\frac{D}{d_j}\right)^2\left[(n'+1) + \frac{d\log(D/d_j)}{d\log(8V/D)}\right]\right\} \quad (5)$$

where

$$K_3 = 4K_1K_2\left(\frac{1}{4} - \frac{2}{\alpha+3} + \frac{1}{2(\alpha+2)}\right)$$

$$K_4 = 2K_2^2\left(\frac{1}{2} - \frac{2}{\alpha+3} + \frac{1}{2(\alpha+2)}\right)$$

The jet method of determination of normal stress difference, has the twin advantages of ease of experimental set up and applicability at high rates of shear. Certain pertinent questions about the validity of the method are admissible.

No energy balance has been made on the jet. The dissipation of mechanical energy is assumed not to affect the jet radius. Any temperature variation due to dissipation is neglected.

The flight distance of the jet before the relaxation of stresses is not known and is a function of the characteristic time of the fluid. It is assumed that the stresses are fully relaxed at the point of measurement of the jet radius.

The Ellis model has sufficient flexibility to fit the data for different fluids in a number of different geometrics, and is an improvement on the power law in the following ways: (*1*) when $\alpha \rightarrow 1$, the fluid behaves like a Newtonian fluid; (*2*) when $(1/\eta_0) \rightarrow 0$, the fluid behaves like a power law fluid; (*3*) the model has a characteristic time given by $(\eta_0/t_{1/2})$, and eq. (5) is therefore a more general equation covering a greater range of fluids.

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